Concentrated Investments, Uncompensated Risk and Hedging Strategies

by

Craig McCann, PhD, CFA and Dengpan Luo, PhD[1]

Investors holding concentrated investments are exposed to *uncompensated* risk – additional risk for which they receive no additional expected return. Investors bearing uncompensated risk are more likely to suffer catastrophic investment losses. Uncompensated risk can be inexpensively estimated and the wealth destroyed by concentrated portfolios can be measured before losses materialize.

It is the responsibility of investment professionals advising clients to explain the uncompensated risk in a portfolio and to counsel against continuing to hold concentrated positions. Investment professionals should also explain widely available risk management strategies. In this paper, we present a general discussion of the risk in concentrated positions and explain and evaluate strategies for reducing the risk of concentrated investments.

Introduction

Prudent investment management requires that securities portfolios be diversified because diversification reduces risk without reducing expected returns. This call to diversify is not new or novel. Modern portfolio theory, developed in the 1950s and 1960s, became best practices in the 1970s and 1980s and was fully embodied in the Prudent Investor Rule and the Uniform Prudent Investor Act by the early 1990s.[2]

Still, many investment portfolios are concentrated in a few stocks. Portfolios may remain concentrated because a stock held is subject to private lockup agreements or regulatory re-sale restrictions, because of tax considerations or because of strongly held views about future returns. Whatever the reason, maintaining concentrated positions destroys wealth and this cost must be factored into any informed investment decision-making.[3]

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 www.slcg.com Dr. McCann can be reached at 703-246-9381 and Dr. Luo can be reached at 703-246-9382.

[2] *See* the Uniform Prudent Investor Act at http://www.law.upenn.edu/bll/ulc/fnact99/1990s/upia94.htm.

[3] We previously wrote about the risk of concentrated stock positions acquired as a result of the exercise of employee stock options and evaluated a particularly perverse strategy promoted for reducing the risk of concentrated investments. *See* "The Suitability of Exercise and Hold," 2002 and "The Use of Leveraged Investments to Diversify a Concentrated Position," 2004. both available at www.slcg.com

Risk, Concentrated Investments, and Wealth Destruction

Risk

Investors care about the risk in their future investment results. An investor might ask, what is the best estimate of the future return to my portfolio or security? What is the range of possible outcomes surrounding that most likely future return? How likely is the return I receive to be much, much less than expected? For some investors, especially those approaching retirement or otherwise living off their investments, the question might be, how likely is my wealth to run out before I run out of needs? We use theory, data and judgment to answer these questions.

There are many ways of defining and measuring investment risk. Three common statistical measures of investment risk are standard deviation, semi-variance and beta. Some investors might more naturally think of risk as the probability of falling short of some target rate of return or level of wealth in the future.[4]

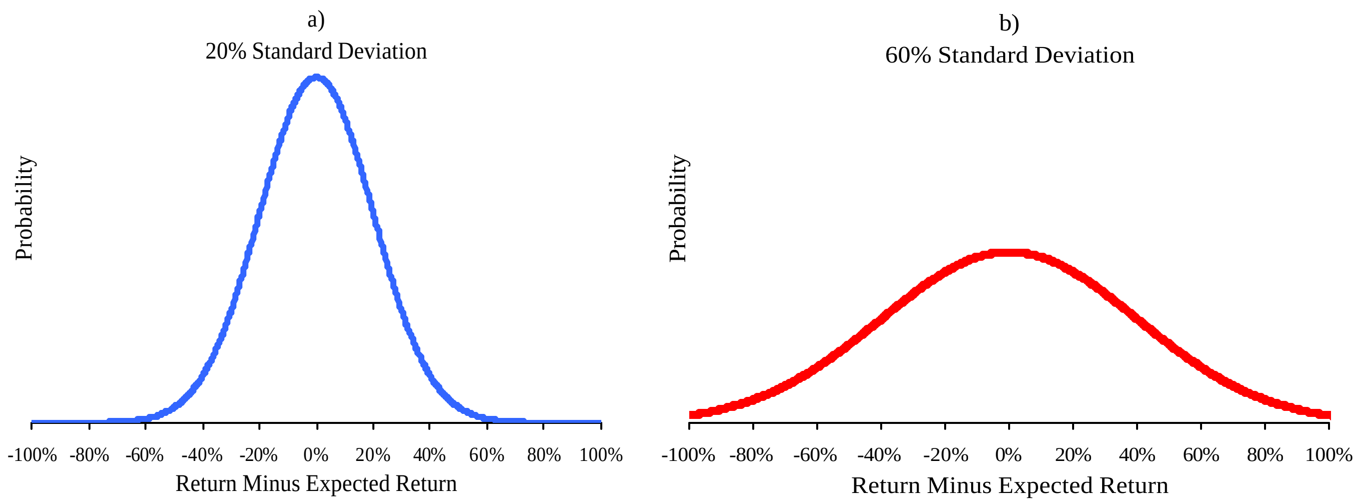
Standard Deviation Captures the Spread in a Bell Curve

The investment risk in a portfolio is typically measured by the standard deviation of the portfolio's returns. The symbol σ is used for standard deviation. Standard deviation captures how observed returns vary around the average or expected return. Standard deviation estimates the "spread" in the distribution using observations that are above as well as below the average value.[5] A portfolio with the same expected return but with a larger standard deviation will be more "spread out." Probability distributions for return series with a 20% standard deviation and with a 60% standard deviation are plotted in Figure 1a) and 1b).

[4] As is standard in the literature, in the next section we will measure risk by the standard deviation of returns. Later when we explore hedged positions with asymmetric payoffs we will measure risk using both standard deviation and semi-variance. We will have little to say about Beta since our focus is on investments that make up substantially all of an individual's securities investments and therefore Beta is of little relevance.

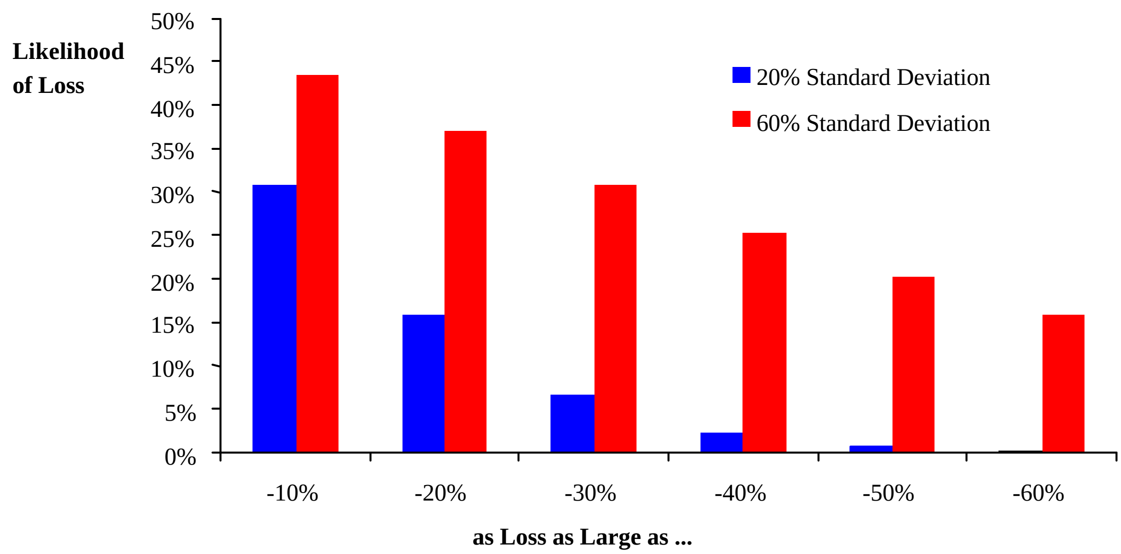
[5] If returns are normally distributed, $^2/_3$rds of the observed values will be within one standard deviation of the average value. For example, if the average or expected return is 12% and the standard deviation is 20%, $^1/_6{}^{th}$ of the observed returns will be below -8%, $^2/_3$rds of the observations will be between -8% and +32%, and $^1/_6{}^{th}$ of the returns will be greater than 32%.

Figure 1
Return Distributions Are More Spread Out for Larger Standard Deviations



Distributions with larger standard deviations are more likely to generate a bad outcome – and more likely to generate a good outcome – than are distributions will smaller standard deviations. The probabilities of observing various below average returns for return series with a 20% standard deviation and with a 60% standard deviation are plotted in Figure 2.

Figure 2
Large Losses Are More Likely the Larger the Standard Deviation



Semi-Variance Focuses on the Downside

Semi-variance is similar to standard deviation but uses only the below average returns to estimate risk. Intuitively this is appealing since investors think of risk as the chance of a bad outcome not a good outcome. Even so, standard deviation is preferable to semi-variance if returns are symmetrically distributed because observed returns above the average return provide useful information about the likelihood of below average returns. If returns are not symmetrically distributed – as in the case of many hedged positions – semi-variance captures risk better than does standard deviation.

Beta is Useful for Estimating Expected Return - Not Risk

The most popularized - and the most misunderstood - measure of risk is Beta. Standard deviation, variance and semi-variance measure the *total* risk in a portfolio or security while Beta measures only the *compensated* (or *market* or *non-diversifiable*) risk in a portfolio or security and ignores the *uncompensated* risk. Poorly diversified portfolios contain a lot diversifiable risk which is not captured by Beta. Beta therefore is not a useful measure of a portfolio's total risk. Beta is, in fact, a much better measure of expected return than it is a measure of risk.

Shortfall Probability is Especially Useful for Retirees

The probability of falling short of some given threshold can sometimes be solved for analytically but the mathematics is both complicated and unintuitive. Monte Carlo simulation is a low-cost, highly flexible tool available to estimate this *shortfall* risk. It can be customized to take into account the facts and circumstances of most investors and most retirement portfolios.[6]

Diversification and the Return for Bearing Risk

Putting Securities Together Averages Risk but Disproportionately Reduces Risk

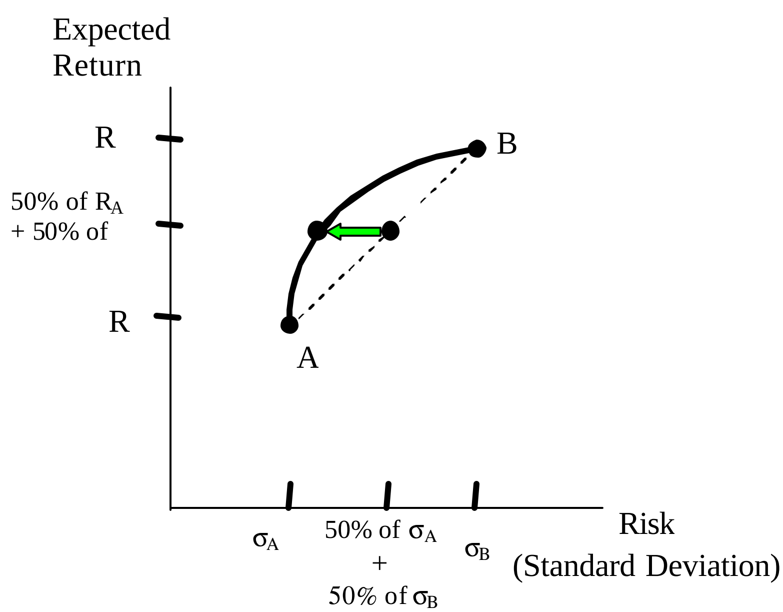
When two or more securities are combined into a portfolio, the resulting portfolio's expected return is equal to the individual securities' expected returns weighted by the fraction of the total portfolio initially invested in each security. If a portfolio is formed by investing half in security A and half in security B, the expected return will equal half of A's expected return plus half of B's expected return. For example, if A's expected return is 12% and B's expected return is 18% the expected return to a 2-security portfolio evenly divided between A and B is 15%.

[6] See "Monte Carlo Simulations," by Craig McCann, PhD and Dengpan Luo, PhD, 2004, forthcoming at www.slcg.com.

While a portfolio's expected return is equal to a weighted-average of the expected returns to the individual securities, (50/50 in our example) the portfolio's *total* risk will be less than the weighted average of the risk of the individual securities if the returns to the individual securities are not perfectly correlated. The returns to stock are never perfectly correlated and so there will be periods during which the return to some securities in a portfolio will be below their average and the returns to other securities in the portfolio are above their average return. When the securities are pooled together, the returns to the portfolio vary less than the returns to the individual securities because the below average returns on some securities are offset by above average returns on other securities.

The combinations of risk and return that can be created using two hypothetical securities are graphed in Figure 3. Expected return is plotted along the vertical axis and risk is plotted along the horizontal axis. The risk and expected return of individual securities and of portfolios of securities are represented as points on the diagram.

Figure 3
Diversification allows Risk to be Reduced
Without Sacrificing Expected Return



If the two securities' returns are perfectly correlated the combinations of risk and return attainable by combining the securities in different proportions trace out a straight line joining the two securities. Along this straight line any reduction in risk is in proportion to the reduction in expected return. For example, if the securities returns are perfectly correlated a portfolio that is half Security A and half Security B will have an expected return equal to the average of the securities' expected returns and will have a risk level equal to the average of the securities' risk levels. If the securities are not perfectly correlated the attainable combinations of risk and return trace out a line that bows toward the left. In this case, a portfolio that is half Security A and half Security B will have an expected return equal to the average of the securities' expected returns but will have a risk level less than the average of the securities' risk levels. The lower the correlation between the securities' returns the greater this disproportionate decrease in risk.

Modern Portfolio Theory

Competitive Pressures Reduce Expected Returns So That Only Non-Diversifiable Risk is Rewarded

Prices of securities are set as a result of investors buying and selling in search of higher returns for bearing risk. Other things equal, investors prefer less risk to more risk and can reduce risk through diversification, so they are willing to pay more for securities held in a diversified portfolio than in isolation. Competitive pressures drive the price of a security up to the point where expected returns just compensate investors for the risk remaining in a security's returns after the security has been included in a well-diversified portfolio. This remaining risk is referred to as non-diversifiable risk. In other words, competitive forces, acting on prices, drive expected returns down to levels that only compensate investors for the non-diversifiable risk in a security.

Figure 4 extends Figure 3 in several ways and contains the familiar modern portfolio theory diagram.[7] Portfolios with the highest expected return for each level of risk trace out the *efficient frontier*. Individual securities and other imperfectly diversified portfolios plot below the efficient frontier. Portfolios along the Capital Allocation Line are created by combining the risky market portfolio with short or long positions in the risk free asset. Portfolios plotting to the left of the market portfolio are invested partly in the market portfolio and partly in the risk free asset. Borrowing and investing more than the portfolio's net value in the market portfolio creates portfolios plotting to the right of the market portfolio. The maximum expected return available for bearing any given level of risk is given by the height of the Capital Allocation Line.[8]

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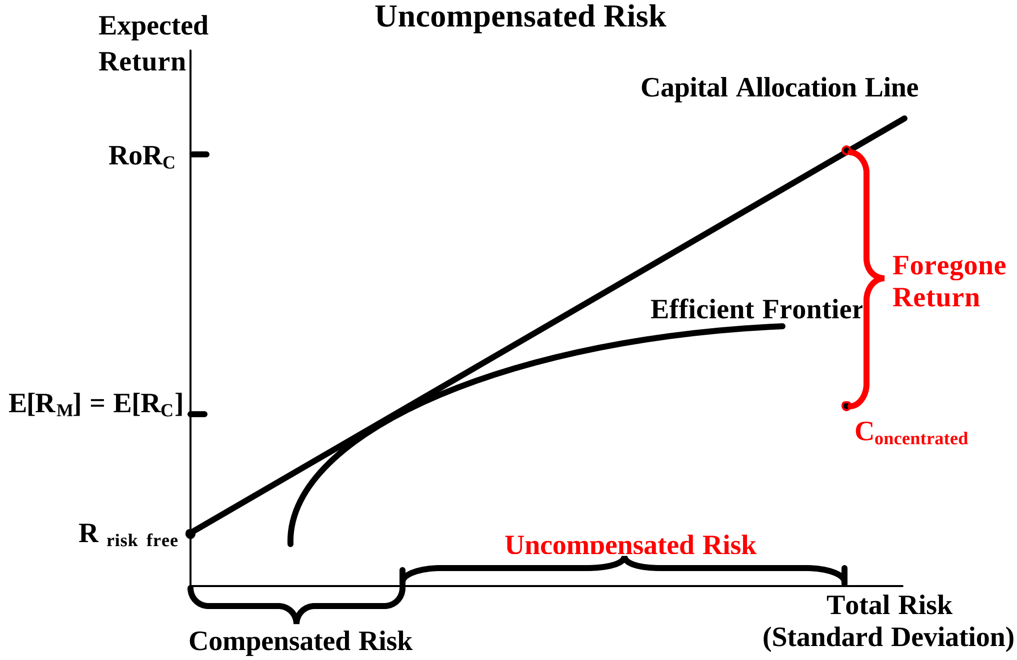
[7] An early application of the Markowitz model to issues of suitability can be found in Stephen B. Cohen "The Suitability Rule and Economic Theory" *Yale Law Journal* (1971) 80:1604-1635.

[8] If there is no risk free borrowing and lending the maximum expected return available for bearing any given level of risk is given by the height of the efficient frontier.

Figure 4
Concentrated Portfolios Contain
Uncompensated Risk



In Figure 4, the concentrated stock position, C, has three times as much risk - but has the same expected return - as the market as a whole. Since someone holding C could have the same expected return with only one-third as much risk, two-thirds of the risk from holding C as a concentrated stock position is *uncompensated* risk.

Uncompensated Risk is a Bet on Red

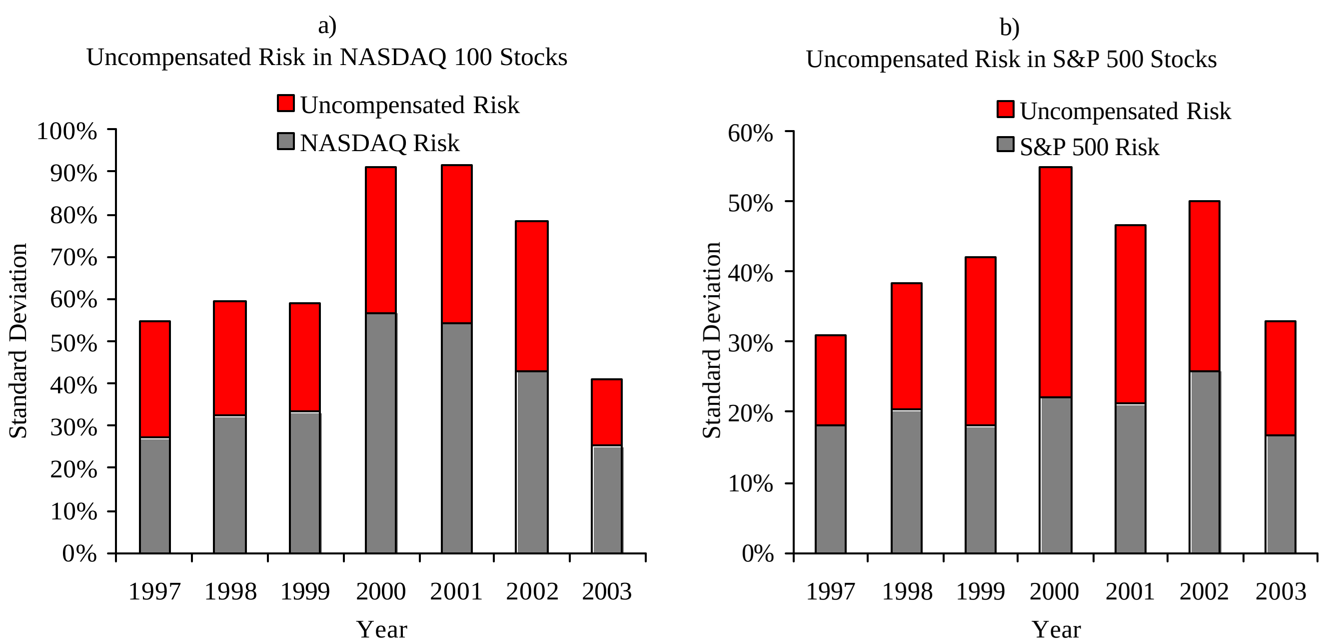
The actual return on an investment in a diversified portfolio of stocks can be much higher or much lower than the expected return. That is, observed returns fluctuate around the expected return. Still, the expected return on a diversified portfolio of stocks is greater than the return on Treasury securities and this difference - *the equity risk premium* - is the compensation investors receive for bearing the risk of stock market fluctuations.

Uncompensated risk on the other hand is like a bet on red; it has a zero expected return. As with the market risk, uncompensated risk can result in large gains or large losses – the difference is that unlike the *equity risk premium* paid as compensation for bearing market risk - the expected return to bearing uncompensated risk is zero. Investors are not compensated with higher expected returns for bearing this non-market risk because it can be diversified away.

The single stock portfolio in Figure 4 is economically equivalent to an investment in the diversified stock market and a separate but equal bet each week on a toss of a fair coin that wins $50,000 if the coin turns up heads and loses $50,000 if the coin turns up tails. Since the expected return on the coin is zero, adding the weekly $50,000 coin toss bet to a diversified investment in the market just adds *uncompensated* risk. Principles of prudent investment management clearly require that this weekly coin toss should be eliminated and so also dictate that diversifiable risk should be eliminated.

We can get a sense of the amount of uncompensated risk in a concentrated stock position by comparing the average risk in individual stocks within an index to the risk of the index itself. During the period from 1997 to 2003, on average an investor holding a concentrated position in a single NASDAQ 100 (or an S&P 500) stock had the same expected return as an investor holding the entire NASDAQ 100 (or the entire S&P 500) but was exposed to nearly twice as much risk on average. *See* Figure 5.

Figure 5
Individual Stocks Contain a Lot of Uncompensated Risk



Prudent Investors Diversify Concentrated Positions

Investors holding concentrated investments are exposed to uncompensated risk. Investors holding well diversified portfolios can achieve higher expected returns than investors holding concentrated positions without bearing any additional total risk by substituting compensated risk for uncompensated risk. That is, as illustrated in Figure 4 above, an investor with two or thee times as much total risk as compensated risk could increase his expected return substantially by diversifying and then reallocating assets so as to achieve the same total risk. Therefore prudent investors diversify away uncompensated risk and then allocate their assets to achieve the greatest expected return for the amount of risk taken on.

Concentration Destroys Investment Value

The failure to diversify away uncompensated risk destroys investment value because the prudently expected rate of return on a concentrated investment is less than the rate of return offered in the marketplace for bearing the same amount of total risk.

The expected future value of an investment is by definition the current value of the investment accumulated at the expected return on the investment. For example, a share of stock selling for $100 today with an expected return of 10% has an expected value of $110 after one year. Also, the current value of an investment is its expected future value discounted to the present at the investment's required rate of return. A share of stock that is expected to be worth $110 in one year is worth $100 today if the required rate of return for bearing the investment's risk for one year is 10%. Together these truisms provide a useful way of estimating the value destroyed as a result of the concentration. The present value of a security will be equal to its current price if the expected rate of return is equal to the required rate or return given the risk of the security.

If the security is held as a concentrated position with a lot of uncompensated risk, the present value of the security will be less than the stock price since the expected rate of return is less than the return investors receive for bearing that much risk. In such cases, the concentrated positions should be sold and the proceeds diversified since doing so allows the investor to recapture some of the wealth destroyed by concentration.

The fraction of wealth destroyed by concentration depends on how much diversifiable risk is in the concentrated position, what the market pays for bearing risk and how long the concentration position will be maintained. The potential wealth destruction from holding concentrated positions is enormous. If we assume that the risk free rate of return is 3%, the equity risk premium is 7%, the expected return of the average stock in the S&P 500 is 10% but the required rate of return for holding the average stock in isolation is 17%. Planning to hold the average NASDAQ stock for one year as a concentrated position is roughly equivalent to throwing away nearly 10% of the value of the investment.

Hedging Strategies

Many strategies have been designed to reduce the risk of a concentrated position. We discuss the structure and effectiveness of the following seven strategies.

1. Covered Calls
2. Protective Puts
3. Cashless Collars
4. Variable Prepaid Forwards
5. Equity Swaps
6. Exchange Funds
7. Leverage Investments

Covered Calls

Call options give investors the right to buy stock at a price no higher than a predetermined price at specified point in the future in exchange for an upfront payment called the call premium. In the covered call writing strategy, an investor who holds a stock sells someone else (the call option buyer) the right to buy the stock in the future at the "strike" price. The option seller (or "writer") is giving up the upside potential in exchange for the receipt of the call option premium. Figure 6 graphs the payoffs to a covered call position.[9]

These payoffs are the same as the payoffs to selling put options and while the investor no longer has the shares of stock the investor does have all the downside below the exercise price of the call options. We demonstrate in an appendix that covered call writing is economically identical to selling all the stock, selling in-the-money puts on the same stock and investing the entire proceeds in Treasury bills. The investor loses dollar-for-dollar every dollar the stock price closes below the strike

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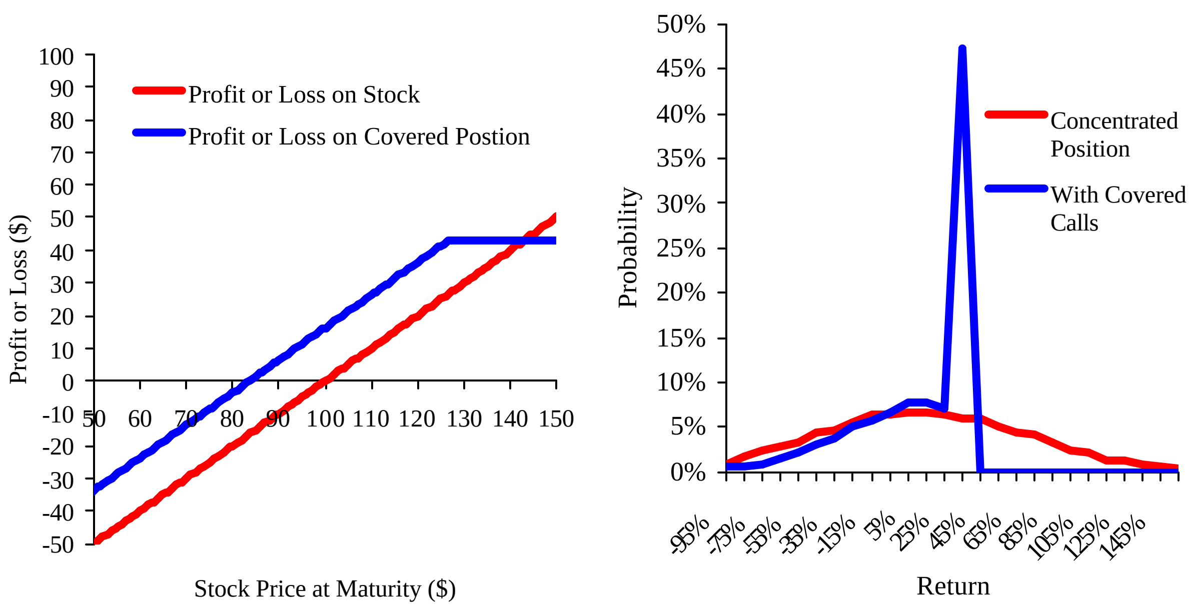
Covered Call Option Example

Initial Stock Price, S_0	\$100
Strike Price, X_c	\$126.70
Stock Return Volatility	60%
Term	1 year
Riskfree Interest Rate	3%
? Black-Scholes Call Value, c_0	\$16.30

price of the call option at expiration. Perversely, this strategy leaves the investor exposed to all the downside in continuing to hold the concentrated position – and more- while eliminating any benefit from increases in the stock price. Often sold as a conservative risk management strategy, covered call writing provides delivers exactly the opposite.

Figure 6
Covered Calls

a) Payoffs at Maturity **b) Probability Distributions**



Protective Puts

Put options give investors the right to sell stock on a future date at a fixed price in exchange for an upfront payment called the put premium. An investor who holds a concentrated position can ensure that he will receive no less than a specified price by purchasing protective put options. The payoffs at expiration to holding put options and holding the underlying security are illustrated in Figure 7.[10]

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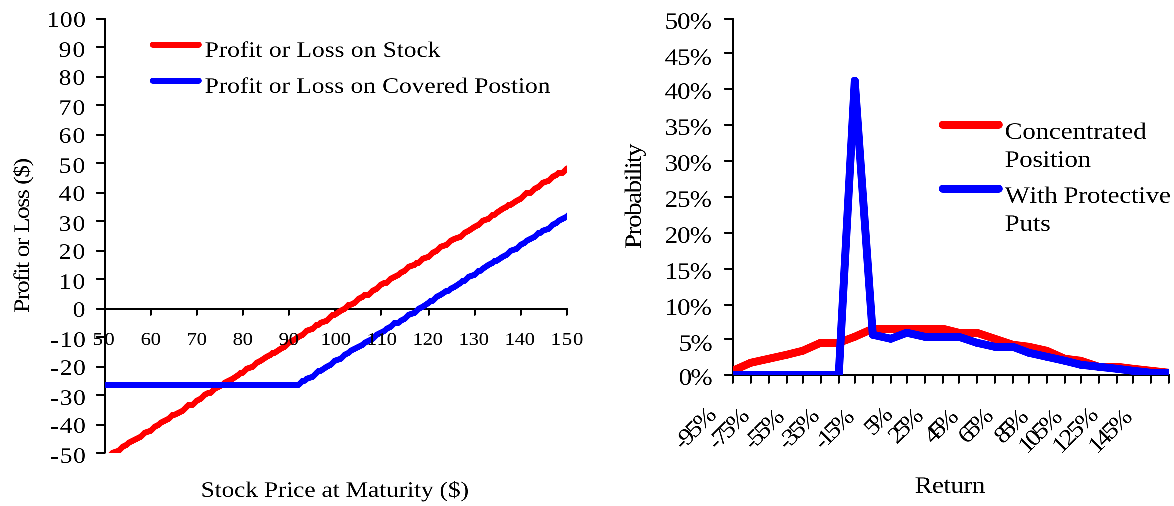
Protective Put Option Example

Current Stock Price, S_0	$100
Put Exercise Price, X_p	$90
Stock Return Volatility	60%
Term	1 year
Riskfree Interest Rate	3%
? Black-Scholes Put Value, p_0	$16.30

Figure 7
Protective Puts

a) Payoffs at Maturity **b) Probability Distributions**



Cashless Collars

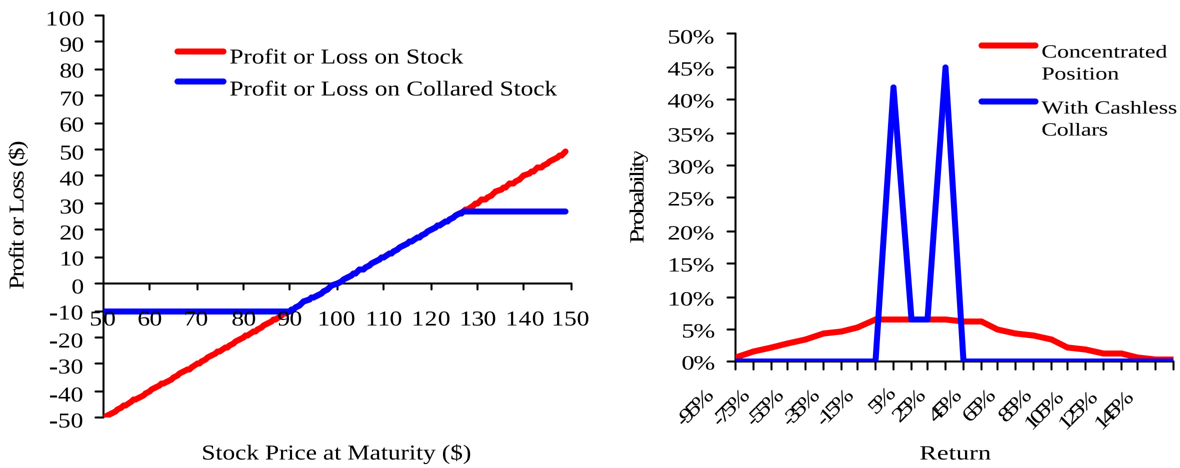
Cashless collars are contracts which have the economic effect of combining covered call options and protective put options. Collars are not typically constructed using exchange traded options but are instead structured as swaps. While there may be tax benefits to packaging collars as over the counter contracts, bundling makes it much more difficult for investors to assess the costs incurred.

Cashless collars work as if call options are sold at strike prices greater than the current stock price and put options are purchased with strike prices below the current stock price. In a cashless collar, the strike prices of the embedded put and call options are adjusted so that the value of the implied call options exceeds value of the implied put options by the transaction costs. The purchaser of the collar limits his downside risk by giving away his upside potential. The greater the downside protection sought (i.e. the higher the implied put option's strike price) the more of the upside potential must be given up (i.e. the lower the implied call option's strike price).

Cashless collars have been customized to include participating collars wherein the investor receives some of the upside potential beyond the implied call option's strike price. This partial participation in the further upside is acquired by accepting a lower strike price on the implied call option or a lower strike price on the implied put option or by partially participating in the downside below the implied put options' strike price. The payoffs at expiration to a cashless collar and holding the underlying security are illustrated in Figure 8.[11]

Figure 8
Cashless Collars

a) Payoffs at Maturity **b) Probability Distributions**



Variable Prepaid Forwards

Variable prepaid forward contracts combine cashless collars with a secured loan. A brokerage firm enters into a cashless collar with the holder of a concentrated position and then lends the investor the value of the number of shares covered by the collar multiplied by the strike price on the put option discounted back from the maturity date to the present at the loan's interest rate.

This structure has been customized to include participating variable prepaid forwards which combine participating collars with an up front loan. The effectiveness of the variable prepaid forward is similar to the effectiveness of a cashless collar.

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Cashless Collar Example

Initial Stock Price, S_0	\$100
Lower Bound, X_p	\$90
Upper Bound, X_c	\$126.7
Stock Return Volatility	60%
Term	1 year
Riskfree Interest Rate	3%
? Black-Scholes Collar Value, $c_0 - p_0$	\$0.00

Equity Swaps

Equity swaps are contracts through which investors exchange the returns to a concentrated position with the returns to a basket or index of securities. For example, an investor holding $1,000,000 of Zoom.inc might agree to swap at periodic intervals the total return to $1,000,000 in Zoom.inc for the total return to the Wilshire 5000 less a fee. The brokerage firm offering the swap can hedge the risk of the swap by trading in Zoom.inc stock or options or by including Zoom.inc in its own portfolio. At maturity, the investor owns $1,000,000 in Zoom.inc at then current market prices plus the return to the benchmark index over the life of the swap.

Exchange Funds

Exchange funds are managed portfolios of pooled contributed investments that provide partial diversification in exchange for a fee and a long term commitment. Exchange funds must hold a leveraged investment in illiquid securities equal to 20% of the value of the contributed securities. Investors contributing securities to an exchange fund each receive an interest in the managed portfolio in proportion to the market value of their contributed securities as a fraction of the combined market value of all the securities contributed to the exchange fund. When investors redeem their interest in the fund they receive their proportionate interest in the net asset value of the fund. In effect, the investors exchange the returns to their contributed security for the returns earned by the exchange fund's managed portfolio. Contributing securities to a qualified exchange fund is not treated as a sale for tax purposes and the exchange fund portfolio will typically be much better diversified than the contributed securities. By contributing securities to an exchange fund, investors are able to achieve improved diversification without realizing a tax liability.

The potential benefit to investors in exchange funds is the ability to gain some diversification immediately without realizing a tax liability. This benefit is greater the greater the amount of unrealized capital gain in the stock. These investors incur significant costs beyond the costs of simply selling the contributed securities. Investors are charged a percent of the value of their contribution as a placement fee. Exchange funds carrying ongoing expenses that are considerably higher than the expense ratios of most mutual funds. Securities contributed to exchange funds typically are highly appreciated positions, since these positions benefit most from tax deferral. These highly appreciated securities also tend to have been more volatile than the average security. Contributed securities tend to be similar to other securities contributed to an exchange fund and so exchange funds' portfolios of contributed securities tend to be closer to a portfolio of NASDAQ stocks than to a portfolio of S&P 500 stocks.

Leveraged Investments

Some brokerage firms and investment advisors had recommended that investors holding a concentrated position in a single stock borrow against it and use the funds to invest in a portfolio of additional stocks to reduce risk. The strategy reduces the variation in returns to the enlarged portfolio of securities but increases the variation in returns to the investor's equity because of the impact of the leverage. This discredited strategy was based on a confused comparison of rates of return on different investment bases. See our Leveraged Diversification paper for a complete discussion of this strategy.[12] The brokerage firms previously advocating this strategy appear to have dropped it, perhaps because of its knowable, demonstrable catastrophic results.

Final Thoughts

Liquidity

The first three strategies – protective puts, covered calls and cashless collars – are directly or indirectly based on options and as such the availability of these strategies is potentially related to the availability of options on the stock to be hedged. There has been a great deal of confusion between the level of trading or open interest in a particular series of option contracts and the *liquidity* of those option contracts. Unlike the market for stock in which market prices adjust so that public investors want to sell exactly the same number of shares that public investors want to buy, options trades are initiated by investors and facilitated by market professionals. In the secondary market for stock, market makers adjust prices to hold de minimus inventories; public investors both initiate and facilitate trades. Options prices are virtually entirely determined by the market price and the volatility of the underlying stock and market professionals accommodate public orders to buy or sell options without searching for another public investor to take the other side of the options trade.

If an investor wishes to buy 600 exchange-traded put option contracts it does not matter at all whether there has been any trading or any open interest in that series. The market maker responsible for the desired option series posts bid and ask quotes and stands ready to transact at those prices up to a specified quote size. The market maker may choose to execute the entire order at the posted quote and must expose the order to floor traders who may offer to execute some or all of the trade between the

[12] *See* "The Use of Leveraged Investments to Diversify a Concentrated Position," by Craig McCann, PhD and Dengpan Luo, PhD, 2004, available at www.slcg.com.

market maker's quotes. If the entire order doesn't get executed immediately the market maker might revise his quotes but his ability to revise the quotes is disciplined by the floor traders and by competition from the other exchanges where the particular option series is traded.

Delta Hedging

In the retail context, applications of the options-based hedging strategies are typically discussed using a constant number of options contracts covering a number of shares less than or equal to the number of shares held by the investor. For example, 1,000 put option contracts covering 100,000 shares with a strike price of $80 might be modeled as a possible hedging strategy for an investor holding 100,000 shares of Zoom.inc currently trading at $100 per share. Such a strategy will typically not be the most effective hedge since the value of the put options will not vary dollar for dollar with the value of the Zoom.inc stock. For example, if Zoom.inc's stock price declines to $90 the value of the put contracts will go up but by less than $10 per share covered under the contract. If the value of the put will only increase by $5 when the stock price falls by $10 we say the options delta is 0.5. To lock in the initial $100 value of the Zoom.inc stock twice as many contracts would need to be purchased as there are shares held.

Appendix A

Put-Call Parity

Put-call parity says that the value of a share of stock and a put option on the stock is equal to the value of the exercise price of the put discounted from the expiration date to the present at the risk free rate of return plus the value of a call option on the same stock with the same strike price and the same expiration. That is, $S_0 + p_0 = \frac{X}{(1+r)^{T-t_0}} + c_0$. Consider the value of combinations of stock and a put option and cash and a call option (with the same strike price and expiration date as the put option) for different possible values of the stock price at the expiration listed below.

Derivation of Put-Call Parity

Stock Price at Expiration	Value of Stock and Put $S_T + p_T$	Value of Cash and Call Option $X + c_T$
$S_T < X$	X	X
$S_T = X$	X	X
$S_T > X$	S_1	S_1

If the stock price is less than the exercise price at expiration the put option is worth the shortfall between the exercise price and the stock price and the call option is worthless. In this case, an investor would exercise the put option, tender the stock and receive \$X. The second investor holding \$X and the call option would discard the worthless call option and - like the first investor - be left with \$X. If the stock price equals X at expiration, the put and the call options both expire worthless and the investor has \$X whether they are holding cash or stock. If the stock price is more than the exercise price at expiration, the put option is worthless and the call option is worth the difference between the stock price and the exercise price. In this case, an investor holding the stock and the put option would throw away the worthless put option and hold onto the stock. The second investor would exercise the call option, tender \$X and - like the first investor - be left with a share of stock.

Since the combination of a share of stock and a put option is equal in value to the combination of the exercise price and a call option with the same exercise price and expiration date as the put option at expiration whatever the ultimate value of the stock, investors are indifferent between these combinations of assets. Investors must also be indifferent between holding the stock with a put option and holding the call option with cash sufficient to accumulate at the risk free rate of interest to the exercise price of the options by expiration. That is, put-call parity holds. We use put call parity to demonstrate the equivalence between covered call writing and holding cash and writing puts. We want to demonstrate the following.

Value $_T$ (Hold Stock, Write Calls) ° Value (Sell Stock, Write Puts).

An investor holding stock who decides to sell a call option with a strike price of X will either have either the stock value or the exercise price – whichever is lower – plus the call premium collected plus interest when the option expires. That is, Value $_T$ (Hold Stock, Write Calls)

$$= \mathrm{Min}\{S_T, X\} + c_0(1+r)^{T-t0}$$

An investor who sells the stock worth S_0 and sells a put option with a strike price of X will have the stock sale proceeds plus the put premium accumulated at the risk free rate but will lose any shortfall in the stock price at expiration below the exercise price of the put. That is, Value $_T$ (Sell Stock, Write Puts) $= (p_0 + S_0)(1+r)^{T-t0} - \mathrm{Max}\{X - S_T, 0\}$.

We can demonstrate the equivalence of the covered call writing and the put writing strategies as follows (put-call parity is used in the first step):

Value (Hold Stock, Write Calls) $= \mathrm{Min}\{S_T, X\} + c_0(1+r)^{T-t0}$

$$= \mathrm{Min}\{S_T, X\} + (p_0 + S_0 - \frac{X}{(1+r)^{T-t0}})(1+r)^{T-t0}$$

$$= \mathrm{Min}\{S_T, X\} - X + (p_0 + S_0)(1+r)^{T-t0}$$

$$= \mathrm{Min}\{S_T - X, 0\} + (p_0 + S_0)(1+r)^{T-t0}$$

$$= -\mathrm{Max}\{X - S_T, 0\} + (p_0 + S_0)(1+r)^{T-t0}$$

= **Value (Sell Stock, Write Puts)**

This is a powerful result. The covered call writing has a capped upside equal to the value of a put with the same exercise price and expiration as the call being sold plus the risk free rate of return on the stock value plus the put premium. This modest *maximum* return is coupled with all the downside risk that was in the stock position - and more since the investor loses dollar-for-dollar for every dollar the stock is lower than the exercise price at expiration.